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Radiative Corrections to the Decay $\tau \rightarrow \pi \nu_\tau$

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Abstract

We have calculated the radiative corrections to the decay $\tau \rightarrow \pi(K) \nu_\tau$, taking to account internal bremsstrahlung and structure dependent radiation in the radiative decay and point meson, hadronic structure dependent and short distance contributions in the virtual corrections. We display the spectra of the photon energy and of the pion-photon invariant mass in the decay $\tau \rightarrow \pi \nu_\tau \gamma$ and compare with the PHOTOS Monte-Carlo. Our result for the radiative correction to the ratio $\Gamma(\tau \rightarrow \pi \nu_\tau(\gamma))/\Gamma(\pi \rightarrow \mu \nu_\mu(\gamma))$ is $\delta R_{\tau/\pi} = (0.16 \pm 0.14) \%$ and for the ratio $\Gamma(\tau \rightarrow K \nu_\tau(\gamma))/\Gamma(K \rightarrow \mu \nu_\mu(\gamma))$, we obtain $\delta R_{\tau/K} = (0.90 \pm 0.22) \%$.

Invited talk at the Third Workshop on Tau Lepton Physics (Montreux, September 1994)

presented by: Markus Finkemeier

1 Introduction

1.1 Lowest Order Prediction

The simplest semihadronic decay mode of the tau lepton is unique in that there is essentially no uncertainty in the theoretical prediction at the order $O(\alpha^0)$. The decay width $\Gamma(\tau \rightarrow \pi \nu_\tau)$ can be predicted using the precisely measured pion decay width $\Gamma(\pi \rightarrow \mu \nu_\mu)$. The only parameter in both decays is the pion decay constant f_π defined by the matrix element of the weak hadronic current,

$$\langle 0 | J_{\text{weak}}^\mu | \pi^+(p) \rangle = i\sqrt{2}f_\pi p^\mu \quad (1)$$

In the ratio $R_{\tau/\pi}$ of the decay rates

$$R_{\tau/\pi} = \frac{\Gamma(\tau \rightarrow \pi \nu_\tau)}{\Gamma(\pi \rightarrow \mu \nu_\mu)} \quad (2)$$

the coupling f_π cancels, and the resulting formula involves the masses of the particles only,

$$R_{\tau/\pi} = \frac{1}{2} \frac{m_\tau^3}{m_\pi m_\mu^2} \frac{(1 - m_\pi^2/m_\tau^2)^2}{(1 - m_\mu^2/m_\pi^2)^2} + O(\alpha) \quad (3)$$

Multiplying this ratio by the experimental decay width $\Gamma(\pi \rightarrow \mu \nu_\mu)$, one gets the $O(\alpha^0)$ prediction for $\Gamma(\tau \rightarrow \pi \nu_\tau)$.

1.2 Radiative Corrections of the order $O(\alpha)$

Because of the infra-red divergences, at the order $O(\alpha)$ one has to consider the ratio of inclusive decay rates into final states with or without an additional photon:

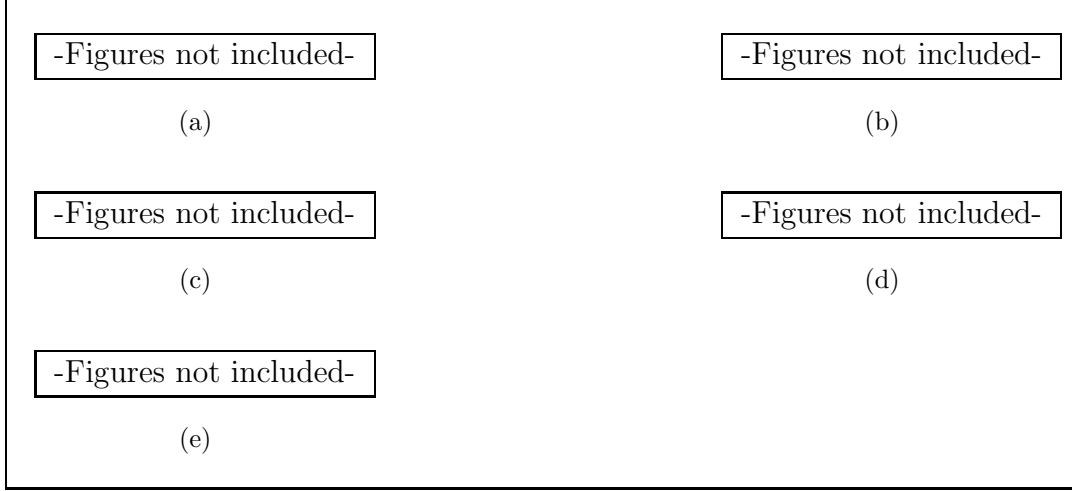
$$R_{\tau/\pi} = \frac{\Gamma(\tau \rightarrow \pi \nu_\tau(\gamma))}{\Gamma(\pi \rightarrow \mu \nu_\mu(\gamma))} = \frac{1}{2} \frac{m_\tau^3}{m_\pi m_\mu^2} \frac{(1 - m_\pi^2/m_\tau^2)^2}{(1 - m_\mu^2/m_\pi^2)^2} (1 + \delta R_{\tau/\pi}) \quad (4)$$

The radiative correction $\delta R_{\tau/\pi}$ arises from the general Feynman diagrams shown in Fig. 1, and the corresponding diagrams for the decay $\pi \rightarrow \mu \nu_\mu(\gamma)$. There are the virtual corrections, Fig. 1 (a)–(c), and the corrections due to the radiative decay, (d)–(e). The diagrams fall into three classes. The diagrams of the first class (Fig. 1 (a) and (d)), where the photon only couples to the leptonic side, are determined by f_π and QED, and so they can be calculated without theoretical uncertainty. The diagrams of the third class (Fig. 1 (c)), where the photon couples twice to the hadronic side, give identical corrections for the tau and pion decay and so cancel in $R_{\tau/\pi}$. Thus the essential problem is to calculate the diagrams of the second class, where the photon couples once to the hadronic side, Fig. 1 (b) and (e).

What makes the calculation of these radiative corrections difficult and interesting is the fact that due to the integration over the momentum of the virtual photon, the physics of very different energy scales Q is involved. For very small energy scales, $Q^2 \ll m_\rho^2$, the relevant amplitudes are fixed by low energy theorems of QCD. For intermediate scales, $Q^2 \approx m_\rho^2$, the amplitudes are dominated by hadronic resonances, and for large scales $Q^2 \gg m_\rho^2$, they are dominated by the short distance behaviour of the weak interaction.

In Sec. 2 we will discuss the radiative decay, which is not only an essential part of the total radiative correction, but also of interest in its own right. In Sec. 3 we will then consider the virtual corrections and the total correction to the ratio $R_{\tau/\pi}$.

Figure 1: Diagrams determining the radiative corrections to $\tau \rightarrow \pi \nu_\tau$



2 The Radiative Decay $\tau \rightarrow \pi \nu_\tau \gamma$

The amplitude for the radiative decay $\tau \rightarrow \pi \nu_\tau \gamma$ can be divided into the so-called internal bremsstrahlung (IB) and the structure dependent radiation (SD) [1, 2, 3, 4, 5]:

$$\mathcal{M}[\tau^-(s) \rightarrow \pi^-(p) \nu_\tau(q) \gamma(k)] = \mathcal{M}_{IB} + \mathcal{M}_{SD} \quad (5)$$

The internal bremsstrahlung corresponds to hooking photons to the external lines, considering the pion as an structureless elementary particle:

$$\mathcal{M}_{IB} = -G_F \cos \theta_c e f_\pi m_\tau \bar{u}_\nu(q) \gamma_+ \left[\frac{p \cdot \epsilon}{p \cdot k} + \frac{\not{k} \not{\epsilon}}{2s \cdot k} - \frac{s \cdot \epsilon}{s \cdot k} \right] u_\tau(s) \quad (6)$$

The structure dependent radiation can be defined as the contribution from the diagram in Fig. 1(e) minus the effective point pion contribution, resulting in

$$\begin{aligned} \mathcal{M}_{SD} = & -\frac{G_F \cos \theta_c}{\sqrt{2}} \left\{ i \epsilon_{\mu\nu\rho\sigma} [\bar{u}_\nu \gamma^\mu \gamma_- u_\tau] \epsilon^\nu k^\rho p^\sigma \frac{F_V(t)}{m_\pi} \right. \\ & \left. + \bar{u}_\nu(q) \gamma_+ [(p \cdot k) \not{\epsilon} - (\epsilon \cdot p) \not{k}] u_\tau(s) \frac{F_A(t)}{m_\pi} \right\} \end{aligned} \quad (7)$$

This amplitude \mathcal{M}_{SD} involves two form factors $F_V(t)$ and $F_A(t)$, which depend on the invariant mass squared of the pion-photon system, $t = (p+k)^2$. The vector and axial vector form factors F_V and F_A correspond to the $J^P = 1^-$ and $J^P = 1^+$ projections of the W boson, respectively. And so in order to be able to calculate the decay rate, we have to parameterize the form factors $F_V(t)$ and $F_A(t)$.

2.1 Normalization at $t = 0$

We use the following values for the form factors at zero momentum transfer:

$$\begin{aligned} F_V(t=0) &= \frac{m_\pi}{4\sqrt{2}\pi^2 f_\pi} = 0.0270 \\ F_A(t=0) &= 0.0116 \pm 0.0016 \end{aligned} \quad (8)$$

Here $F_V(0)$ is obtained from the axial anomaly, whereas $F_A(0)$ has been measured in the radiative pion decay $\pi \rightarrow e\nu_e\gamma$ [6].

However, for the radiative tau decay, we need F_V and F_A in the whole range $m_\pi^2 \leq t \leq m_\tau^2$. In order to parameterize the energy dependence of the form factors we assume vector and axial vector meson dominance.

2.2 Parametrization of $F_A(t)$

In the case of the axial form factor, the only established resonance which has the correct quantum numbers is the $a_1(1260)$. Thus we assume

$$F_A(t) = \text{BW}_{a_1}(t) F_A(0) \quad (9)$$

where in the normalized Breit Wigner resonance factor

$$\text{BW}_{a_1}(t) = \frac{m_{a_1}^2}{m_{a_1}^2 - t - im_{a_1}\Gamma_{a_1}(t)} \quad (10)$$

we use an energy dependent width $\Gamma_{a_1}(t)$ as calculated from the $a_1 \rightarrow 3\pi$ and $a_1 \rightarrow \rho\pi$ phase space [7].

2.3 Parametrization of F_V

In the case of the vector form factor, there are three resonances with the correct quantum numbers in the mass range covered by the tau, viz. the $\rho = \rho(770)$, $\rho' = \rho(1450)$ and the $\rho'' = \rho(1700)$. And so the vector meson dominance ansatz for $F_V(t)$ reads

$$F_V(t) = \frac{F_V(0)}{1 + \lambda + \mu} [\text{BW}_\rho(t) + \lambda \text{BW}_{\rho'}(t) + \mu \text{BW}_{\rho''}(t)] \quad (11)$$

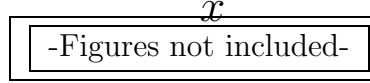
We have to fix the two parameters μ and λ which determined the size of the contribution of the higher radial resonances. Noting that $F_V(t)$ is related by CVC (conserved vector current) to the form factor $F_{\pi\gamma\gamma^*}(t)$ which determines the coupling $\pi^0\gamma\gamma^*(q^2 = t)$ [8], we are able to find four constraints for the two parameters:

1. The limit of $F_{\pi\gamma\gamma^*}$ for large negative t is predicted by perturbative QCD [9]

$$\lim_{t \rightarrow -\infty} t F_{\pi\gamma\gamma^*}(t) = 2f_\pi \quad (12)$$

2. The slope of $F_{\pi\gamma\gamma^*}(t)$ at $t = 0$ has been measured in $\pi^0 \rightarrow e^+e^-\gamma$ [6].

Figure 2: Photon energy spectrum of the decay $\tau \rightarrow \pi \nu_\tau \gamma$: Total prediction (solid), pure internal bremsstrahlung (dashed) and the prediction from TAUOLA + PHOTOS (dots with statistical error bars)



3. The value $F_V(m_\rho^2)$ of the vector form factor at the rho mass is related to the decay width $\Gamma(\rho \rightarrow \pi \gamma)$ which has been measured.
4. Similarly, $F_V(m_{\rho'}^2)$ is related to $\Gamma(\rho' \rightarrow \pi \gamma)$, which has not been measured but is in turn related by vector meson dominance to the measured width $\Gamma(\rho' \rightarrow \pi \omega)$.

The parameter choice

$$\lambda = 0.136; \quad \mu = -0.051 \quad (13)$$

simultaneously fulfills all four constraints.

2.4 Numerical Results

Having obtained parameterizations for the hadronic structure dependent form factors, we can calculate decay distributions for $\tau \rightarrow \pi \nu_\tau \gamma$ [4]. In Fig. 2 we display the photon energy spectrum $d\Gamma/dx$ of the decay. The dimensionless parameter x is defined by

$$x = \frac{2E_\gamma}{m_\tau} \quad (14)$$

where E_γ is the photon energy in the tau rest frame. In Fig. 2, we do not only display the total spectrum, which we predict, but also the contribution from pure internal bremsstrahlung (IB) (i.e. neglecting hadronic structure dependent effects) and the result from the Monte-Carlo program PHOTOS [10]. PHOTOS provides a semiclassical approximation of the IB contribution. We find from Fig. 2 that PHOTOS gives a very good approximation to the IB part. Furthermore we find that the IB is strongly dominating the total spectrum, except for the region of very hard photons where the rate is extremely small anyway.

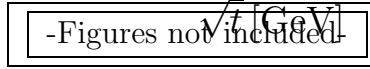
So in fact we find that PHOTOS gives a very good approximation to the photon spectrum of $\tau \rightarrow \pi \nu_\tau \gamma$, and that hadronic structure dependent effects are almost invisible in this spectrum.

A observable much better suited to separate the structure dependent effects from the internal bremsstrahlung is the pion-photon invariant mass spectrum $d\Gamma/dm_{\pi\gamma}$, which we display in Fig. ???. The dimensionless parameter z is defined by

$$z = \frac{m_{\pi\gamma}^2}{m_\tau^2} \quad (15)$$

In this spectrum, a clear ρ resonance peak and a smaller a_1 peak are visible.

Figure 3: Pion-photon invariant mass spectrum of the decay $\tau \rightarrow \pi\nu_\tau\gamma$: Internal bremsstrahlung (IB, dashed), structure dependent radiation (SD, dash-dotted), interference of IB and SD (INT, dotted), and the total spectrum (solid)



A measurement of this spectrum would be very interesting both in order to check our model and in order to have another channel to measure the poorly known width Γ_{a_1} . However, there is a very large background from the decay $\tau \rightarrow \pi^- \pi^0 \nu_\tau$, where the π^0 decays into two photons and one of these may escape detection.

It is interesting to mention the total size of the hadronic structure dependent contribution (integrated over the full phase space),

$$\frac{\Gamma_{SD+INT}(\tau \rightarrow \pi\nu_\tau\gamma)}{\Gamma_0(\tau \rightarrow \pi\nu_\tau)} = (0.11 - 0.06)\% = 0.05\% \quad (16)$$

where the first number (0.11) corresponds to the structure dependent amplitude squared (SD) and the second one (-0.06) to the interference between structure dependent radiation and internal bremsstrahlung (INT).

3 Virtual Corrections

Calculating the virtual corrections, we have to perform loop integrations over the Euclidean photon momentum k_E^2 from $k_E^2 = 0$ up to $k_E^2 = m_Z^2$. In order to be able to perform this integration, we separate the integration range into two parts [11, 12, 13]. In the long distance region of $k_E^2 = 0 \cdots \mu_{cut}^2 \approx O(1 \text{ GeV}^2)$, mesons are the relevant degrees of freedom. For this region we build a realistic phenomenological model, taking into account low energy theorems of QCD, assuming vector meson dominance and considering experimental constraints such as the electromagnetic form factor of the pion, which is known quite well in the relevant momentum region. For the short distance integration, $k_E^2 = \mu_{cut}^2 \cdots m_Z^2$, we use the parton model.

3.1 Long Distance Part

In the integration over small photon momenta, $k_E^2 = 0 \cdots \mu_{cut}^2$, we proceed as follows. We start from the diagrams which correspond to the lowest order, $O(P^2)$, of low energy QCD (effective point pion [14], see Fig. 4). We then modify these diagrams to account for the momentum dependence of the electromagnetic form factor of the pion, which is known to be dominated by rho-like resonances, see Fig. 5. We then add to this the loop diagrams which correspond to the structure dependent (SD) part in the radiative decays, see Fig. 6. As indicated in the figure, we assume a double vector meson dominance of the relevant form factors H_V and H_A ,

$$H_V(k, p) = \text{BW}_\omega(k^2) F_V[(k + p)^2]$$

Figure 4: Effective Point pion diagrams

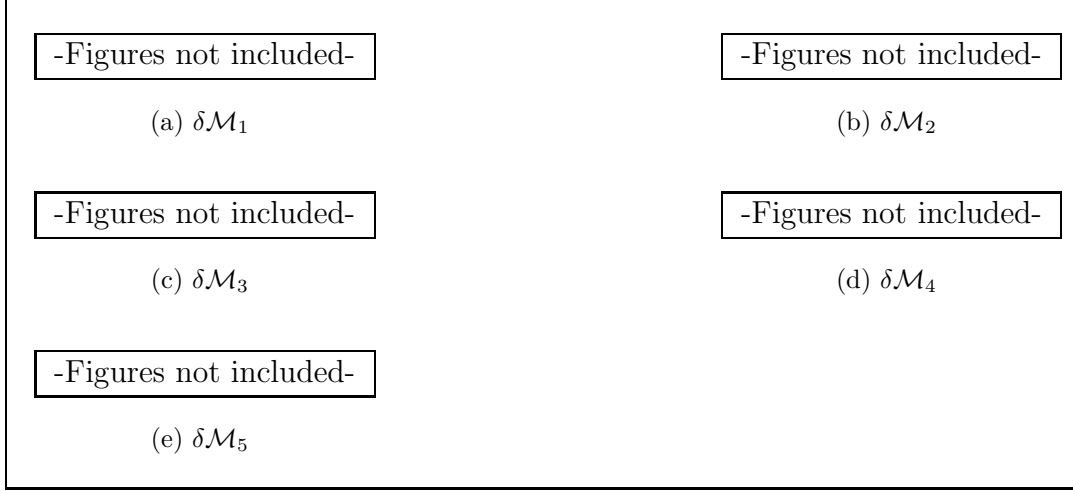
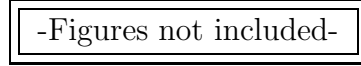


Figure 5: Vector meson dominance of coupling of the photon to the pion. In fact we include the ρ , the ρ' and the ρ'' with relative strengths which have been obtained by fitting the measured pion electromagnetic form factor [7].



$$H_A(k, p) = \text{BW}_\rho(k^2) F_A[(k + p)^2] \quad (17)$$

where k is the momentum of the virtual photon, p the pion momentum, and F_V and F_A are the form factors involved in the radiative decays (real photons). In order to estimate the model dependence, we also compare to a form with a single vector meson dominance,

$$\begin{aligned} H_V(k, p) &= F_V[(k + p)^2] \\ H_A(k, p) &= F_A[(k + p)^2] \end{aligned} \quad (18)$$

Figure 6: Hadronic structure dependent loop diagrams

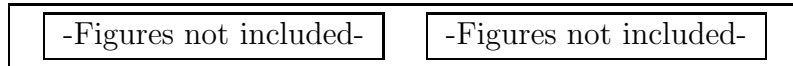
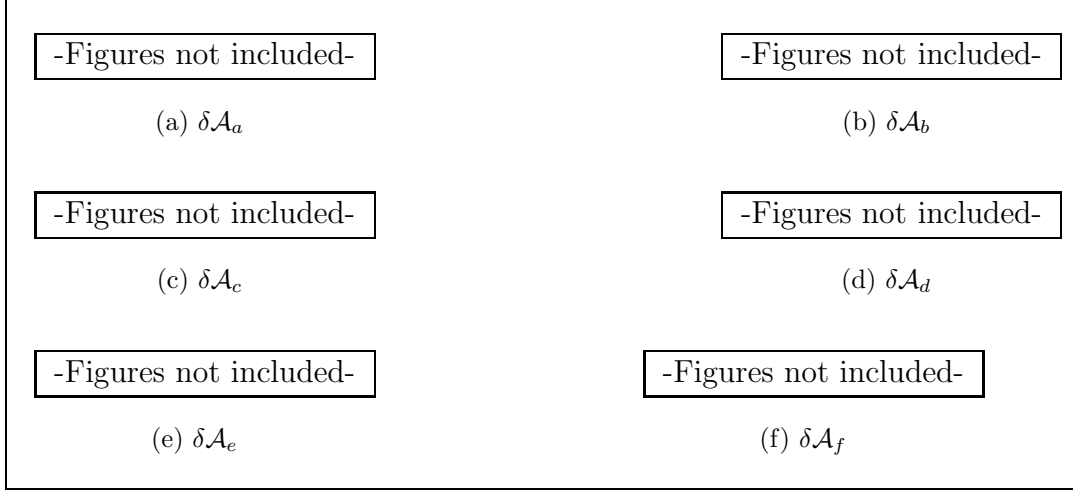


Figure 7: Short distance diagrams



3.2 Short Distance Corrections

For large virtual photon momenta, $k_E^2 = \mu_{cut}^2 \cdots m_Z^2$, we calculate the corrections to the elementary vertex $\tau \rightarrow \nu_\tau \bar{u} d$, see Fig. 7, and plug the result into Fig. 8. Doing the same thing for the pion decay, we obtain the short distance contribution to the radiative correction to the ratio $R_{\tau/\pi}$ in the form

$$\left(\delta R_{\tau/\pi}\right)_{sd} = \frac{3}{2f_\pi} \int_{-1}^{+1} du \Phi_\pi(u) r(u) \quad (19)$$

The integration extends over the scaled relative momentum of the two quarks in the infinite momentum frame. $\Phi_\pi(u)$ is an unknown parton distribution function, and $r(u)$ has been calculated from the short distance diagrams. $r(u)$ varies only very little over the integration range, and so we can approximate it by its value at $u = 0$,

$$\left(\delta R_{\tau/\pi}\right)_{sd} \approx r(0) \frac{3}{2f_\pi} \int_{-1}^{+1} du \Phi_\pi(u) = r(0) \quad (20)$$

where last equation follows from a sum rule [15]. In fact we find that the short distance correction is dominated by a leading logarithmic contribution [13, 16],

$$\left(\delta R_{\tau/\pi}\right)_{sd} \approx \frac{2\alpha}{\pi} \frac{m_\tau^2}{m_\tau^2 - \mu_{cut}^2} \ln \frac{\mu_{cut}}{m_\tau} \quad (21)$$

3.3 Numerical Results

Adding up long and short distance corrections, we obtain the total radiative correction, which depends on the choice of the matching scale μ_{cut} and on the choice of the parameters of the

Figure 8: Decay $\tau \rightarrow \pi\nu_\tau$ via an intermediate quark-antiquark state. The bubble on the left hand side is to be replaced by the short distance diagrams $\delta\mathcal{A}_i$ of Fig. 7

-Figures not included-

hadronics. In Fig. 3.3 we display the radiative correction to the width $\tau \rightarrow \pi\nu_\tau$ in variation with μ_{cut} , using three different sets for the hadronic parameters. There is a standard set (I) with some central values, and sets (II) and (III) with reasonable variations around these central values. While for the sets (I) and (III) the dependence of the correction of the matching scale μ_{cut} is reasonably small, it is unacceptably large for (II). In fact, (I) and (III) use the double vector meson dominance parameterization of (17), (II), however, uses the single pole form of (18), which therefore can be excluded.

We obtain similar results for the radiative correction to $\Gamma(\pi \rightarrow \mu\nu_\mu)$. Taking the ratio, we find

$$\delta R_{\tau/\pi} = (0.16 \pm 0.08 \pm 0.04 \pm 0.07)\% = (0.16 \pm 0.14)\% \quad (22)$$

The first error quoted (0.08%) arises from the matching uncertainties in $\Gamma(\tau \rightarrow \pi\nu_\tau(\gamma))$, the second one from the matching in $\Gamma(\pi \rightarrow \mu\nu_\mu(\gamma))$, and the last one from the uncertainties in the hadronic parameters. The number given here for $\delta R_{\tau/\pi}$, $(0.16 \pm 0.14)\%$, differs slightly from the number we quoted in [13], $\delta R_{\tau/\pi} = (0.16^{+0.09}_{-0.14})\%$. This is due to the fact that in [13], we estimated the matching uncertainty by considering the variation of the ratio $R_{\tau/\pi}$, whereas now we have discussed the matching uncertainties of the individual decay rates. The latter approach should give a better estimate of the true model dependence, because in the ratio scale dependences associated with a mismatch of long and short distances tend to cancel at large scales.

Note that the rather small total radiative correction arises from cancellations of larger numbers with opposite signs.

Similarly, we obtain the radiative correction to the ratio of the tau decay into a kaon $\Gamma(\tau \rightarrow K\nu_\tau(\gamma))$ and the decay width of the kaon $\Gamma(K \rightarrow \mu\nu_\mu(\gamma))$

$$\delta R_{\tau/K} = (0.90 \pm 0.08 \pm 0.09 \pm 0.14)\% = (0.90 \pm 0.22)\% \quad (23)$$

Our result for $R_{\tau/\pi}$ should be compared with the recent estimate by Marciano and Sirlin [17], which in terms of $R_{\tau/\pi}$ reads

$$\delta R_{\tau/\pi} = (0.67 \pm 1.)\% \quad (24)$$

where the $\pm 1.\%$ is the authors' estimate of the missing long distance corrections to the tau decay rate, which they did not calculate. And so we confirm their result within their estimated error bars, but we are able to reduce the error substantially by performing a complete calculation. Translating the radiative corrections into predictions for the branching ratios, we

Figure 9: Radiative correction to $\Gamma(\tau \rightarrow \pi\nu_\tau)$, using different choices for the parameters of the hadronic structure dependent correction: Standard choice (I) (solid), and variations (II) and (III) (dashed and dotted, respectively)

$$\delta \frac{\Gamma}{\Gamma_0}(\tau \rightarrow \pi\nu_\tau(\gamma))[\%]$$

-Figures not included-

$$\mu_{cut} [\text{GeV}]$$

get

$$\begin{aligned} \text{BR}(\tau \rightarrow \pi\nu_\tau(\gamma)) &= (10.946 + \overbrace{0.005}^{\text{SD} + \text{INT}} \pm 0.020)\% \times \left(\frac{\tau_\tau}{291.6 \text{ fs}} \right) \\ \text{BR}(\tau \rightarrow K\nu_\tau(\gamma)) &= (0.723 + \overbrace{0.002}^{\text{SD} + \text{INT}} \pm 0.004)\% \times \left(\frac{\tau_\tau}{291.6 \text{ fs}} \right) \\ \text{BR}(\tau \rightarrow h\nu_\tau(\gamma)) &= (11.669 + \overbrace{0.007}^{\text{SD} + \text{INT}} \pm 0.021)\% \times \left(\frac{\tau_\tau}{291.6 \text{ fs}} \right) \end{aligned} \quad (25)$$

where h denotes the inclusive sum of pions and kaons. Note that here we have separated out the contribution SD + INT associated with structure dependent radiation, corresponding to the decay chains $\tau \rightarrow \rho\nu_\tau$, $\rho \rightarrow \pi\gamma$ and $\tau \rightarrow a_1\nu_\tau$, $a_1 \rightarrow \pi\gamma$. Whereas we have included all photons in $\delta R_{\tau/\pi}$, experimental numbers for the branching ratios do not include hard photons. Note, however, that the size of this SD + INT part is in all three cases small compared to the overall uncertainty of the prediction. Using the new world average for the tau lifetime [18]

$$\tau_\tau = (291.6 \pm 1.6) \text{ fs} \quad (26)$$

we obtain

$$\text{BR}_\pi^{theo} = (10.95 \pm 0.06)\%$$

$$\begin{aligned}
\text{BR}_K^{theo} &= (0.723 \pm 0.006)\% \\
\text{BR}_h^{theo} &= (11.67 \pm 0.06)\%
\end{aligned}
\tag{27}$$

These predictions agree within one standard deviation with the new world averages [19] quoted at this conference.

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